

LEARNING OBJECTIVES:

- 2.03.01 Identify the five general types of radiation measurement errors.
- 2.03.02 Describe the effect of each source of error on radiation measurements.
- 2.03.03 State the two purposes for statistical analysis of count room operations.
- 2.03.04 Define the terms "mean," "median" and "mode."
- 2.03.05 Given a series of data, calculate mean, median, or mode.
- 2.03.06 Define the terms "variance" and "standard deviation."
- 2.03.07 Given the formula and required information, calculate the standard deviation.
- 2.03.08 Explain how a Chi square value is used in analysis of count room measurements.
- 2.03.09 Given the formula, perform Chi-square analysis on given count room data, and state whether the data has the expected amount of randomness.
- 2.03.10 Explain the meaning of counting data reported as $x \pm y$ dpm.
- 2.03.11 Given counting results and appropriate formulas, report results to desired confidence level.
- 2.03.12 List and explain the methods used to improve the statistical validity of count room measurements.
- 2.03.13 Given counting and source data, calculate efficiencies and correction factors.

INTRODUCTION

Radiological sample analysis involves observation of radioactive decay, a random process, and estimation of the amount of radioactive material present based on that observation. Activity measurements are used to make decisions that may affect the health and safety of workers at those facilities and their surrounding environments.

OVERVIEW

This unit presents an overview of measurement processes, and statistical evaluation of both measurements and equipment performance. In addition, this

unit addresses some of the actions to take to minimize sources of error in count room operations.

2.03.01 GENERAL SOURCES OF ERROR

Assuming a counting system is calibrated correctly, there are five general sources of error associated with counting a sample (cpm) and deducing its activity (dpm).

- a. Self-absorption
- b. Backscatter
- c. Resolving time
- d. Geometry
- e. Random disintegration of radioactive atoms (statistical variations).

Items a, b, c, and d are errors associated with the counting system and are called "systematic errors". Item e is called a "statistical error".

2.03.02. EFFECTS OF ERRORS ON RADIATION MEASUREMENTS

a. Self-Absorption. Self-absorption is the absorption of the emitted radiation by the sample material. Self-absorption could occur for:

- Liquid samples with a high solid content
- Air samples from a high dust area
- Use of the improper filter paper may introduce a type of self-absorption, especially in alpha counting (absorption by the media, or filter).

When a sample has an abnormally high amount of material on the sample media, it could introduce an error due to self-absorption. Personnel collecting samples should ensure the correct sample media are used and that the sample does not become too heavily loaded with sample. If the sample is thick, counts will be lost, so the measured value will be less than the actual value.

b. Backscatter

Backscatter occurs when the emitted radiation traveling away from the detector is reflected, or scattered back, by the material in back of the sample. The amount of radiation that is scattered back into the detector will depend on the type and energy of the radiation and the type of backing material (reflector). The amount of backscattered radiation increases as the energy of the radiation increases and as the atomic number of the backing material increases. Generally, the backscatter error is only a consideration for particulate radiation, such as alpha and beta particles. Because beta particles are more penetrating than alpha particles, the backscatter error will be more pronounced for beta particles. The ratio of measured activity of a beta source counted with a reflector compared to counting the same source without a reflector is called the backscatter factor.

Normally, the backscatter error is accounted for in the efficiency or conversion factor of the counting system. However, if different reflector materials, such as aluminum and stainless steel are used in calibration and operation, an additional unaccounted error is introduced. This additional error will be about 6% for stainless steel versus aluminum. Count room personnel must be aware of the reflector material used during calibration of the counting equipment. Any deviation from that reflector material will introduce an unaccounted error and reduce confidence in the analysis results.

c. Resolving Time

Resolving time is the time interval which must elapse after a detector pulse is counted before another pulse can be counted. Any radiation entering the detector during the resolving time will not be recorded as a pulse; therefore, the information on that radiation interaction is lost. As the activity, or decay rate, of the samples increases, the amount of information lost during the resolving time of the detector is increased. As the losses from resolving time increase, an additional error in the measurement is introduced.

Count room personnel should be aware of the limitations for sample count rate based on procedures and the type detector in use to prevent the introduction of additional resolving time losses. This is especially true for counting equipment that uses GM detectors.

d. Geometry

Geometry of the sample and detector describes the positioning of the sample in relation to the detector. Radiation is emitted from a sample equally in all directions. Normally, only a fraction of the emitted radiation is emitted in the direction of the detector because the detector does not surround the sample. If the distance between the sample and the detector is varied, then the fraction of emitted radiation which hits the detector will change. This fraction will also change if the orientation of the sample under the detector (side-to-side) is varied. An error in the measurement can be introduced if the geometry of the sample and detector is varied from the geometry used during instrument calibration. This is especially critical for alpha counting where any change in the sample to detector distance also increases or decreases the chance of shielding the alpha particles by the air between the sample and detector. Examples of this kind of error are:

- Piling smears and/or filters on top of each other in the same sample holder. Piling of the samples moves the top sample closer to the detector and varies the calibration geometry.
- Using deeper or shallower sample holders than those used during calibration changes the sample to detector distance.
- Adjusting movable bases in the counting equipment sliding drawer changes the sample to detector distance.

- Using too many or not using the appropriate sample holder or planchet changes the sample to detector distance.
- Plexiglass shelving in counting chamber is improperly set.

e. **Random Disintegration**

The fifth source of general counting error is the random disintegration of the radioactive atoms and will be discussed in the remainder of the lesson.

2.03.03. STATISTICS

Statistics is defined in Webster's as "a branch of mathematics that deals with the organization, analysis, collection, and interpretation of statistical data." Webster's defines a statistic as an estimate of a variable, as an average or a mean, made on the basis of a sample taken from a larger set of data.

This last definition is applicable to our discussion of counting statistics. After all, when we take samples, we use the data derived from analysis of those samples to make determinations about conditions in an area, in water, or in air, etc., assuming that the sample is representative.

So, we have estimated conditions (a variable) on the basis of a sample (our smear, water sample, air sample) taken from a larger set of data.

Now, to study these processes, we can use proven statistical models to evaluate our observations for error.

Application of Statistical Models

Application of specific statistical methods and models to counting operations is termed counting statistics and is essentially used to do two things.

- Predict the inherent statistical uncertainty associated with a measurement, thus allowing us to estimate the precision associated with that measurement. (See sections 6 and 7)
- Serve as a check on the normal function of nuclear counting equipment. (See sections 8 and 9)

2.03.04. MODE, MEDIAN AND MEAN

Mode - An individual data point that is repeated most in a particular data set.

- Example: In a set of test scores, a 95 score occurs (is repeated) more often than any other score.

Median - The center value in a data set arranged in ascending order.

•Example: In the same set of test scores as above, this is the score in the middle where one half the scores are below and the other half are above the median.

Mean - Average value of all the values in the data set.

2.03.05. Determination of mode, median, and mean

Determination of the mode of a particular data set requires no real calculation, just observation. In the data set containing 1.8, 1.8, 1.9, 1.9, 2.0, 2.0, 2.0, 2.0, 2.0, 2.2, 2.3, 2.3, 2.3, 2.5, observation reveals that 2.0 appears more often than any other measurement, thus, 2.0 is the mode.

Determination of the median first requires arrangement of individual data points into ascending order. The median is then the centerpoint. In the data set used above, both 7th and 8th data points are 2.0, thus the median is 2.0. If the 7th and 8th data points were different, the median would be the average of the two values.

Determination of the mean requires that all data points in the data set be added and the sum divided by the number of data points in the set. In the set used above, the sum is 29 and there were 14 data points, thus the mean is 2.07.

- Mean determination is often expressed using special symbols
- Mean is represented by an x with a bar on top of it: \bar{x}
- The number of data points is represented by a small n
- "Sum of" is represented by the symbol Σ
- Mean = $\frac{\Sigma x}{n}$

2.03.06. VARIANCE AND STANDARD DEVIATION

Both the variance and the standard deviation are measures of the amount of scatter of individual data points around the mean. Mathematically, the standard deviation is the square root of the variance.

2.03.07. CALCULATION OF THE STANDARD DEVIATION

The standard deviation is also defined mathematically as:

$$\sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

where:

- x = Sample counts for each data point
- \bar{x} = Mean
- (n-1) = Number of data points less one

If most of the data points are located close to the mean, the curve will be tall and steep and have a low numerical value for a standard deviation.

If data points are scattered, the curve will be lower and not as steep and have a larger numerical value for a standard deviation.

In a Gaussian (or "normal") distribution, it has been determined mathematically that 68.2% of the area under the curve falls within the data points located at the mean \pm one standard deviation; 95.4% of the area under the curve falls between the data points located at \pm two standard deviations.

- 68.2% of the time the observed successes, or counts, will be within ± 1 standard deviation of the mean
- 95.4% of the time the observed successes, or counts, will be within ± 2 standard deviations of the mean
- 99.7% of the time the observed successes, or counts will be within ± 3 standard deviations of the mean.

The known statistical distribution is used when setting up a piece of counting room equipment and in evaluating its operation by means of a daily source check.

As a part of equipment setup, a check source is counted a specified number of times for a specified count time. Following the completion of this series of counts (trials) the mean and standard deviation are calculated.

Sample Calculation #1. Given the following set of data, calculate the mean and sample st

$$\text{Mean} = \bar{x} = 1429/10 = 142.9$$

$$\text{Variance} = 1062.9/(10-1) = 118.1$$

$$\text{Standard deviation} = \text{squareroot of variance} = 10.9$$

Many scientific calculators will calculate the standard deviation. Some have two versions, the "sample" standard deviation s , and the population standard deviation σ (sigma). The version used in this lesson is the sample standard deviation, s . If you plan to use your calculator, make sure you know which version to use.

2.03.08. CHI-SQUARE

Using the same data, a Chi square analysis is performed. This answers the question of how closely the observed distribution of data about the mean agrees with the expected distribution, to verify the presence of the expected amount of randomness in the data set. The Chi-square numerical value is compared to the range of values given in a table for the specified number of trials. If the value is too low, it tells us that there is not

a sufficient degree of randomness in the observed data. If the value is too high, it tells us that there is too much randomness in the observed data.

2.03.09. CHI-SQUARE CALCULATION

In principle, the Chi square test compares the observed with the expected variance. The formula to calculate chi-square is:

$$\frac{\sum(x - \bar{x})^2}{\bar{x}}$$

The numerator is (n-1) times the observed variance. Using the formulae for counting statistics (Poisson statistics) we can show that the expected variance is equal to the number of counts \bar{x} , and the expected standard deviation is therefore $\sqrt{\bar{x}}$. The formula for chi-square is therefore (n-1) times the ratio of the observed to the expected variance.

Sample Calculation #2. Using the data from sample calculation # 1 above, perform Chi square analysis.

$$\frac{\sum(x - \bar{x})^2}{\bar{x}} = \frac{1062.9}{142.9} = 7.4$$

The value is close to (n-1), indicating that the observed and expected variances are in reasonable agreement.

The Chi square test is often referred to as a "goodness of fit" test. Chi square is answering the question: how well does this set of data fit a normal distribution curve? If it does not fit a curve indicating sufficient randomness, then the counting instrument may be malfunctioning.

Exercises on mean, median, mode, variance, standard deviation and chi-square

1. a) If you shake a handful of coins, how many would you expect to come up heads? This value is the "mode" for a large number of attempts n . If you try it once, would you be surprised if you sometimes obtained a different number?
b) The following numbers were obtained by shaking 10 coins and counting the number that came up heads; then repeating ten times: 7,5,4,3,5,4,4,7,4,6
Calculate the **mean, median, mode, variance, standard deviation, and chi-square** of these 10 numbers.
2. a) If you shake a pair of dice, what is the most probable score, ie the mode of a large number of attempts? If you try this, would you be surprised if your score was different?
b) The following numbers were obtained by shaking two dice, and repeating ten times: 8,10,9,5,9,6,5,6,3,9. Calculate the **mean, median, mode, variance, standard deviation, and chi-square** of these 10 numbers.
3. The following set of data is the result of an experiment like exercise 1, but performed by a suspicious looking character using her own set of 10 special coins:
Number coming up heads: 5,6,5,4,5,5,6,4,5,5.
Calculate the **mean, median, mode, variance, standard deviation, and chi-square** of these 10 numbers. Comment on the results.
4. This same suspicious character produces a pair of dice. Results for 10 throws are:
12,10,12,7,2,8,10,12,6,11.
Calculate the **mean, median, mode, variance, standard deviation, and chi-square** of these 10 numbers. Comment on the results.
5. The following numbers came from an ESP-1 counting events for ten seconds:
9,10,6,13,11,10,13,7,11,10
Calculate the **mean, median, mode, variance, standard deviation, and chi-square** of these 10 numbers.

Answers:

1. a) mode for a large number of attempts is 5. If the mode of a small sample is not 5, this is not surprising.
b) mean=4.9, median between 4 and 5, mode=4, variance=1.88, standard deviation = 1.37, chi-square=16.9/4.9=3.4
2. a) 7, no. b) mean=7, median between 6 and 8, mode=9, variance=5.3, standard deviation = 2.3, chi-square=48/7=6.9
3. mean=5.0, median=5.0, mode=5.0, variance=0.44, standard deviation=0.67, chi-square=4/5=0.8. Four of the coins have heads on both sides and another four have tails on both sides.
4. mean=9, median=10, mode=12, variance=10.7, standard deviation=3.3, chi-square=96/9=10.7. The dice are weighted so 6 comes up most often.
5. mean=10.0, median=10, mode=10, variance=5.1, standard deviation=2.26, chi-square=46/10=4.6

Expected amount of randomness

If you take a smear sample and count the same smear three times, will you (a) get exactly the same number every time, or (b) will the measurements be distributed about the mean with a spread corresponding to the variance?

(Answer: b.)

You can get the actual observed variance using the equations of section 2.03.06 and 2.03.07. You can also predict the expected variance. As discussed above, if the measurements have the expected amount of randomness, the variance will be about the same as the mean value, usually within a factor of 2, almost certainly within a factor of 10.

This can be used to estimate whether the expected amount of randomness is present in several counts of the same sample. The chi-square is $(n-1)$ times $(\text{variance})/(\text{mean})$. If the ratio $(\text{variance})/(\text{mean})$ is between 0.1 and 10 then the measurements have the expected amount of randomness.

For example if the three measurements of the sample count are: first count: 110; second count: 120; third count: 100; you can guess that the standard deviation is approximately the deviation from the mean to most of the individual data, i.e. about 10. Or you can use the formula and show that the standard deviation is exactly 10. The variance is $10^2 = 100$.

variance = 100

mean = 110

$(\text{variance})/(\text{mean}) = 100/110 = 0.9$

This is between 0.1 and 10, so the data have the expected amount of randomness.

Exercises.

An RCT counts the same smear three times. In each of the following cases, so the data sets show the expected amount of randomness, too much randomness, or too little randomness?

1. {100, 101, 100}
2. {100, 110, 225}
3. {100, 112, 88}
4. {400, 440, 420}

Answers:

1. variance = 0.33, mean = 100.3, ratio = 0.003, not enough randomness
2. variance = 4825, mean = 145, ratio = 33.3, too much randomness
3. variance = 144, mean = 100, ratio = 1.44, expected amount of randomness
4. variance = 400, mean = 420, ratio = 0.95, expected amount of randomness

2.03.10. ERROR CALCULATIONS

The error present in a measurement governed by a statistical model can be calculated using known parameters of that model.

Radiation laboratories are expected to operate at a high degree of precision and accuracy. However, since we know that there is some error in our measurements, we are tasked with reporting measurements to outside agencies in a format that identifies that potential error. The format that is used specifies the activity units and a range in which the number must fall.

For example, the results of an activity measurement would be reported as a given activity x plus or minus the deviation, or error, y , in the measurement: $x \pm y$ dpm. This means that our measurements indicate the activity as x dpm, which is the reported value. However, it could be as little as $x-y$ dpm or as much as $x+y$ dpm.

2.03.11. CONFIDENCE LEVEL

Since radiation laboratories prefer to be right more than they are wrong, counting results are usually reported in a range that would be correct 95% of the time, or at a 95% confidence level.

The error is some multiplier times the standard deviation. The multiplier to use is based on the confidence level that is desired, and is derived from the area under the Gaussian distribution curve.

Common multipliers used include:

<u>Confidence Level</u>	<u>Multiplier</u>
68%	1.0
90%	1.645
95%	1.960

To calculate the range to the point at which you would expect to be right 95% of the time, you would multiply the standard deviation times 1.96 and report the results of the measurement as x dpm \pm 1.96 standard deviations. Note that using a 68% confidence level introduces an expected error a large percentage of the time. Therefore, for reasonable accuracy a higher confidence level may be used.

BACKGROUND

The error present in a measurement includes the error present in the gross count, which contains both sample and background, and the error present in the background count.

Rules for propagation of error preclude merely adding the two errors together.

The total error in the measurement is calculated by squaring the error in the background and adding that to the square of the error in the gross count and taking the square root of the sum.

Mathematically, this is written:

$$E = \sqrt{E_g^2 + E_b^2}$$

Where:

E = Error present in the measurement

E_g = Error in count of sample plus background (gross count)

E_b = Error in count of background

Since we normally report this in terms of count rates, R_g and R_b, the formula is slightly modified as follows:

$$E = \sqrt{\frac{R_g}{t_g} + \frac{R_b}{t_b}}$$

Sample calculation #3. The counting rate for a sample was 250 cpm.

Assume 10 minute counting time and zero background counts.

Report sample activity at a 95% C.L.

$$\text{standard deviation} = \sqrt{\frac{R}{t}} = \sqrt{\frac{250}{10}} = 5 \text{ cpm}$$

For 95% C.L., multiply 5 cpm by 1.96 to get 10 cpm (approximately).

Sample activity = 250 ± 10 cpm at 95% C.L.

Sample calculation #4. A long lived sample is counted for one minute and gives a total (gross) of 562 counts. A one minute background gives 62 counts.

Report results to 95% CL.

count rate = 562 - 62 = 500 cpm.

$$\text{standard deviation} = \sqrt{\frac{R_g}{t_g} + \frac{R_b}{t_b}} = \sqrt{\frac{562}{1} + \frac{62}{1}} = 25 \text{ cpm}$$

For 95% C.L., multiply 25 cpm by 1.96 to get 50 cpm (approximately).

Sample activity = 500 ± 50 cpm at 95% C.L.

Exercises on background.

1. A sample is counted for 1 minute and a gross count rate of 350 cpm is observed. The background is also counted for 1 minute and the rate is 30 cpm. Calculate the sample count rate, and give the result in the format $x \pm y$ @95%C.L.

2. A sample is counted for 1 minute and a gross count rate of 34 cpm is observed. The background is also counted for 1 minute and the rate is 30 cpm. Calculate the sample count rate, and give the result in the format $x \pm y$ @95%C.L.

3. The same sample is counted for 16 minutes at a gross count rate of 34 cpm. The background is counted for 1 minute and the rate is 30 cpm. Calculate the sample count rate, and give the result in the format $x \pm y$ @95%C.L.
4. The same sample is counted for 16 minutes at a gross count rate of 34 cpm, and the background is also counted for 16 minutes at 30 cpm. Calculate the sample count rate, and give the result in the format $x \pm y$ @95%C.L.
5. The background is reduced, so the gross count rate is 16 cpm and the background is 12 cpm. Both are counted for 16 min. Calculate the sample count rate, and give the result in the format $x \pm y$ @95%C.L.
6. To get a smaller error we may count for longer. If the gross count rate is 16 cpm, the background is 12 cpm, and both are counted for one hour, calculate the sample count rate, and give the result in the format $x \pm y$ @95%C.L. NOTE that the time, 1 hour, must be multiplied by 60 to convert to minutes.

Answers

1. 320 ± 38 cpm @95%CL
2. 4 ± 16 cpm @95%CL
3. 4 ± 11 cpm @95%CL
4. 4 ± 4 cpm @95%CL
5. 4.0 ± 2.6 cpm @95%CL
6. 4.0 ± 1.3 cpm @95%CL

2.03.12. Improving the statistical error of count room measurements

If we look at the variables present in the calculation, we can see that we have varying degrees of control over these variables.

$$E = \sqrt{\frac{R_g}{t_g} + \frac{R_b}{t_b}}$$

- **R_g** is the gross sample count rate. We really have no control over this.
- **R_b** is the background count rate. We do have some control over this and the background on any counting equipment should be maintained as low as possible.
- **t_b** is the background counting time and **t_g** is the sample counting time. These are factors that we have control over. To reduce our error we can count a sample and the background for as long a period as we need to.

By minimizing the statistical error present, we improve the validity of our measurements.

2.03.13. EFFICIENCY and CORRECTION FACTORS

A detector intercepts and registers only a fraction of the particles emitted by a radioactive source. The major factors determining the fraction of particles emitted by a source that actuate a detector include:

- The fraction of particles emitted by the source which travel in the direction of the detector window
- The fraction emitted in the direction of the detector window which actually reach the window
- The fraction of particles incident on the window which actually pass through the window and produce ionization

Formulas. Efficiency is the ratio of counts per minute to the number of disintegrations per minute.

$$\text{Efficiency} = \frac{\text{counts per minute (cpm)}}{\text{disintegrations per minute (dpm)}}$$

The efficiency obtained in the formula above will be in fractional (decimal) form. To calculate percent efficiency, the fraction is multiplied by 100.

To obtain the true number of particles emitted per minute, the instrument reading in counts per minute is multiplied by a correction factor.

$$\text{Correction Factor} = \frac{\text{disintegrations per minute (dpm)}}{\text{counts per minute (cpm)}}$$

The correction factor is the inverse of the efficiency and the efficiency is the inverse of the correction factor.

Example 1:

An instrument reads 150,000 cpm when exposed to a 400,000 dpm source. What is the efficiency?

$$\text{Efficiency} = \frac{\text{cpm}}{\text{dpm}}$$

$$\text{Efficiency} = \frac{150,000}{400,000}$$

$$\text{Efficiency} = 0.375 \text{ or } 37.5\%$$

Example 2:

An instrument has an efficiency of 18%. What is the correction factor?

$$\text{Correction Factor} = \frac{1}{\text{Efficiency}} = \frac{1}{0.18} = 5.6$$

Problems.

Solve the following:

1. An instrument reads 3,000 cpm when exposed to a 10,000 dpm Cs-137 calibration source. What is the efficiency?
2. What is the correction factor if the source activity is 8,000 dpm and the instrument reads 400 cpm?
3. An instrument reads 5,000 cpm with a 300 cpm background. The source activity is 92,000 dpm. What is the instrument efficiency?
4. What is the correction factor for an instrument with a 20% efficiency?
5. An instrument reads 11,000 cpm in a background of 250 cpm when measuring a source of 50,000 dpm. What is the efficiency?
6. If the efficiency of a counting instrument is 8%, what would the reading (cpm) be on an 18,000 dpm source?

Answers.

1. 30%
2. 20
3. 5.1%
4. 5
5. 21.5%
6. 1440 cpm

SUMMARY

This unit addressed the measures used to minimize error and the fundamentals of binomial statistics and application of these fundamentals in a nuclear counting environment.

APPENDIX: MINIMUM DETECTABLE ACTIVITY

This section is not mandated by the DOE, and will not be on the exam. Nevertheless the concept of Minimum Detectable Activity (MDA) or Lower Limit of Detection (LLD) is becoming increasingly important. Some students may find this section too brief to follow easily; the diagram suggested at the end of the section may help.

The MDA is defined for a particular counting system as the sample activity that will be correctly identified 95% of the time, ie if the activity is truly present we should get a false negative 5% of the time, and if there is truly no activity we should get a false positive 5% of the time.

Using the notation of sections 11 and 12, the gross count rate R_g has to be measured in the presence of a background rate R_b . We define a decision level at the count rate R_d so that if the count rate is greater than this we conclude that the sample is radioactive, but if the rate is less than this we conclude that we are measuring only background.

If the sample is not radioactive, and we take a large number of measurements, then the measured values of R_g will be distributed about the background rate R_b with a standard deviation s , ie $R_b \pm s$. We choose a decision level: $R_d = R_b + 1.65s$. Recall from section 11 that 90%CL corresponds to 1.65 s . So 90% of the readings will be within the interval from $(R_b - 1.65s)$ to R_d , 5% will be below $(R_b - 1.65s)$ and 5% will be above R_d , ie there will be a false positive 5% of the time. (Draw a sketch to illustrate this.)

The MDA is the sample activity which gives a rate $R_{mda} = R_d + 1.65s$ ie $R_b + 3.30s$ since $R_d = R_b + 1.65s$. If the sample truly has an activity equal to the MDA then 90% of the readings will be in the interval from (R_d) to $(R_{mda} + 1.65s)$ and only 5% will be false negatives, below the decision level R_d .

Suggested diagram: Draw two overlapping Gaussian distributions, each with standard deviation s , one centered at R_b and the other centered at R_{mda} ; mark R_d where they overlap, half way between R_b and R_{mda} ; and shade the regions that contain the false positives and the false negatives.

In summary, we calculate the standard deviation using the formula:

$$s = \sqrt{\frac{R_b}{t_g} + \frac{R_b}{t_b}}$$

which is the formula from sections 11 and 12, assuming that $R_g = R_b$, ie assuming that the true activity is small. Then the MDA is that activity which gives a rate $R_{mda} = 3.3s$.

The formula given by Golnick (p368 at the end of chapter 10) assumes that $t_g = t_b = t$ so that $s = (2R/t)$, ie Golnick's factor $4.66 = 3.3 \sqrt{2}$.